Special Practice Problems

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~[JEE (Mains & Advanced)]~

Topics: T.R.I , Trigo.Equation , Inverse Trigo. Function

Interesting facts in maths:~

1. 259 × 39 × आपकी उम्र = तीन बार आपकी उम्र.

2. 2520 ऐसा सबसे छोटा अंक है जो 1 से लेकर 10 तक के सभी अंको से पूरा divide हो सकता हैं.

Objective Questions Type I [Only one correct answer]

In each of the questions below, four choices are given of which only are is correct. You have to select the correct answer which is the most appropriate.

1. If $k_1 = \tan 27\theta - \tan\theta$ and $k_2 = \frac{\sin\theta}{\cos 3\theta} + \frac{\sin 3\theta}{\cos 9\theta} + \frac{\sin 9\theta}{\cos 27\theta}$, then

(a) $k_1 = 2k_2$ (b) $k_1 = k_2 + 4$

(c) $k_1 = k_2$ (d) none of these

2. If $x = \sin\theta|\sin\theta|$, $y = \cos\theta|\cos\theta|$, where $\frac{99\pi}{2} \le \theta \le 50\pi$, then

(a) $x - y = 1$ (b) $x + y = -1$

(c) $x + y = 1$ (d) $y - x = 1$

3. If $0 < x < \frac{\pi}{2}$ and $\sin^n x + \cos^n x \ge 1$, then

(a) $n \in [2, \infty)$ (b) $n \in (-\infty, 2]$

(c) $n \in [-1, 1]$ (d) none of these

4. If $\frac{\sec^8\theta}{a} + \frac{\tan^8\theta}{b} = \frac{1}{a+b}$, then for every real value of $\sin^2\theta$

(a) $ab \le 0$ (b) $ab \ge 0$

(c) $a + b = 0$ (d) none of these

5. In an acute angled ΔABC the least value of $\sec A + \sec B + \sec C$ is

(a) 6 (b) 8

(c) 3 (d) none of these

6. If $P_n = \cos^n\theta + \sin^n\theta$, then $P_n - P_{n-2} = kP_{n-4}$, where

(a) $k = 1$ (b) $k = -\sin^2\theta\cos^2\theta$

(c) $k = \sin^2\theta$ (d) $k = \cos^2\theta$

7. If $\frac{\cos x}{a} = \frac{\cos(x+\theta)}{b} = \frac{\cos(x+2\theta)}{c} = \frac{\cos(x+3\theta)}{d}$, then $\frac{a+c}{b+d}$ is equal to

(a) a/d (b) c/d

(c) b/c (d) d/a

8. If $U_n = \sin n\theta \sec^n\theta$, $V_n = \cos n\theta \sec^n\theta \ne 1$, then $\frac{V_n - V_{n-1}}{U_{n-1}} + \frac{1}{n}\frac{U_n}{V_n}$ is equal to

(a) 0 (b) $\tan\theta$

(c) $-\tan\theta + \frac{\tan n\theta}{n}$ (d) $\tan\theta + \frac{\tan n\theta}{n}$

9. If $\sin\left(x + \frac{4\pi}{9}\right) = a$; $\frac{\pi}{9} < x < \frac{\pi}{3}$, then $\cos\left(x + \frac{7\pi}{9}\right)$ equals

(a) $\frac{\sqrt{(1-a^2)} - a\sqrt{3}}{2}$ (b) $\frac{1 - a^2 + a\sqrt{3}}{2}$

(c) $\frac{a\sqrt{3} - \sqrt{(1-a^2)}}{2}$ (d) $\frac{-\sqrt{(1-a^2)} - a\sqrt{3}}{2}$

10. If $a = \sin\frac{\pi}{18}\sin\frac{5\pi}{18}\sin\frac{7\pi}{18}$, and x is the solution of the equation $y = 2[x] + 2$ and $y = 3[x-2]$, where $[x]$ denotes the integral part of x, then a is equal to

(a) $[x]$ (b) $\frac{1}{[x]}$

(c) $2[x]$ (d) $[x]^2$

11. If $A = \cos(\cos x) + \sin(\cos x)$ the least and greatest value of A are

(a) 0 and 2 (b) -1 and 1

(c) $-\sqrt{2}$ and $\sqrt{2}$ (d) 0 and $\sqrt{2}$

12. If in a ΔABC, $\tan A + \tan B + \tan C > 0$, then

(a) Δ is always obtuse angled triangle

(b) Δ is always equilateral triangle

(c) Δ is always acute angled triangle

(d) nothing can be said about the type of triangle

13. Let n be a fixed positive integer such that $\sin\frac{\pi}{2n} + \cos\frac{\pi}{2n} = \frac{\sqrt{n}}{2}$, then

(a) $n = 4$ (b) $n = 5$

(c) $n = 6$ (d) none of these

14. In a quadrilateral if
$\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)+\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)=2$,
then $\Sigma \cos\frac{A}{2}\cos\frac{B}{2}$ is equal to

(a) 0 (b) 6
(c) 3 (d) 2

15. If the mapping $f(x)=ax+b, a<0$ maps $[-1,1]$ onto $[0,2]$, then for all values of θ, $A=\cos^2\theta+\sin^4\theta$ is such that

(a) $f\left(\frac{1}{4}\right)\le A\le f(0)$ (b) $f(0)\le A\le f(-2)$
(c) $f\left(\frac{1}{3}\right)\le A\le f(0)$ (d) $f(-1)< A\le f(-2)$

16. If $\tan\alpha, \tan\beta, \tan\gamma$ are the roots of the equation $x^3-px^2-r=0$, then the value of $(1+\tan^2\alpha)(1+\tan^2\beta)(1+\tan^2\gamma)$ is equal to

(a) $(p-r)^2$ (b) $1+(p-r)^2$
(c) $1-(p-r)^2$ (d) none of these

17. If $\alpha,\beta,\gamma,\delta$ are four solutions of the equation $\tan\left(\theta+\frac{\pi}{4}\right)=3\tan 3\theta$, then $\tan\alpha\tan\beta\tan\gamma\tan\delta$ equals

(a) 3 (b) 1/3
(c) $-1/3$ (d) none of these

18. The least value of $\text{cosec}^2 x+25\sec^2 x$ is

(a) 0 (b) 26
(c) 28 (d) 36

19. If $x\sin a+y\sin 2a+z\sin 3a=\sin 4a$
$x\sin b+y\sin 2b+z\sin 3b=\sin 4b$,
$x\sin c+y\sin 2c+z\sin 3c=\sin 4c$.
Then the roots of the equation
$t^3-\left(\frac{z}{2}\right)t^2-\left(\frac{y+2}{4}\right)t+\left(\frac{z-x}{8}\right)=0$, $a, b, c\ne n\pi$, are

(a) $\sin a, \sin b, \sin c$ (b) $\cos a, \cos b, \cos c$
(c) $\sin 2a, \sin 2b, \sin 2c$ (d) $\cos 2a, \cos 2b, \cos 2c$

20. If $x=r\sin\theta\cos\phi$, $y=r\sin\theta\sin\phi$ and $z=r\cos\theta$, then the value of $x^2+y^2+z^2$ is independent of

(a) θ,ϕ (b) r,θ
(c) r,ϕ (d) r

21. If $0°<\theta<180°$, then $\sqrt{2+\sqrt{2+\sqrt{2+\ldots+\sqrt{2(1+\cos\theta)}}}}$, then being n number of 2's, is equal to

(a) $2\cos\left(\frac{\theta}{2^n}\right)$ (b) $2\cos\left(\frac{\theta}{2^{n-1}}\right)$
(c) $2\cos\left(\frac{\theta}{2^{n+1}}\right)$ (d) none of these

22. If $\tan\alpha/2$ and $\tan\beta/2$ are the roots of the equation $8x^2-26x+15=0$, then $\cos(\alpha+\beta)$ is equal to

(a) $-\frac{627}{725}$ (b) $\frac{627}{725}$
(c) -1 (d) none of these

23. If $a\sec\alpha-c\tan\alpha=d$ and $b\sec\alpha+d\tan\alpha=c$ then

(a) $a^2+c^2=b^2+d^2$ (b) $a^2+d^2=b^2+c^2$
(c) $a^2+b^2=c^2+d^2$ (d) $ab=cd$

24. Let n be an odd integer. If $\sin n\theta=\sum_{r=0}^{n} b_r\sin^r\theta$ for all real θ, then

(a) $b_0=1, b_1=3$ (b) $b_0=0, b_1=n$
(c) $b_0=-1, b_1=n$ (d) $b_0=0, b_1=n^2-3n-3$

25. If θ is an acute angle and $\tan\theta=\frac{1}{\sqrt{7}}$, then the value of $\frac{\text{cosec}^2\theta-\sec^2\theta}{\text{cosec}^2\theta+\sec^2\theta}$ is

(a) 3/4 (b) 1/2
(c) 2 (d) 5/4

26. $\tan 7\frac{1}{2}^\circ$ is equal to

(a) $\frac{2\sqrt{2}-(1+\sqrt{3})}{\sqrt{3}-1}$ (b) $\frac{1+\sqrt{3}}{1-\sqrt{3}}$
(c) $\frac{1}{\sqrt{3}}+\sqrt{3}$ (d) $2\sqrt{2}+\sqrt{3}$

27. The maximum value of $\sin(x+\pi/6)+\cos(x+\pi/6)$ in the interval $(0,\pi/2)$ is attained at

(a) $\pi/12$ (b) $\pi/6$
(c) $\pi/3$ (d) $\pi/2$

28. The minimum value of the expression $\sin\alpha+\sin\beta+\sin\gamma$, where α,β,γ are real numbers satisfying $\alpha+\beta+\gamma=\pi$ is

(a) + ve (b) − ve
(c) zero (d) −3

29. If $\sin\alpha=\sin\beta$ and $\cos\alpha=\cos\beta$, then

(a) $\sin\left(\frac{\alpha+\beta}{2}\right)=0$ (b) $\cos\left(\frac{\alpha+\beta}{2}\right)=0$
(c) $\sin\left(\frac{\alpha-\beta}{2}\right)=0$ (d) $\cos\left(\frac{\alpha-\beta}{2}\right)=0$

30. $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$ is equal to

(a) 1/2 (b) $\cos\pi/8$
(c) 1/8 (d) $\frac{1+\sqrt{2}}{2\sqrt{2}}$

31. If $A+C=B$, then $\tan A\tan B\tan C$ is equal to

(a) $\tan A+\tan B+\tan C$ (b) $\tan B-\tan C-\tan A$
(c) $\tan A+\tan C-\tan B$ (d) $-(\tan A\tan B+\tan C)$

32. If A lies in the third quadrant and $3\tan A-4=0$, then $5\sin 2A+3\sin A+4\cos A$ is equal to

(a) 0 (b) $-\frac{24}{5}$
(c) $\frac{24}{5}$ (d) $\frac{48}{5}$

33. If $1+\sin\theta+\sin^2\theta+\ldots\infty=4+2\sqrt{3}$, $0<\theta<\pi, \theta\neq\frac{\pi}{2}$, then

(a) $\theta=\frac{\pi}{6}$ (b) $\theta=\frac{\pi}{3}$

(c) $\theta=\frac{\pi}{3}$ or $\frac{\pi}{6}$ (d) $\theta=\frac{\pi}{3}$ or $\frac{2\pi}{3}$

34. If $\tan(\pi\cos\theta)=\cot(\pi\sin\theta)$, then the value of $\cos\left(\theta-\frac{\pi}{4}\right)$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{2\sqrt{2}}$ (d) none of these

35. The equation $a\sin x+b\cos x=c$, where $|c|>\sqrt{(a^2+b^2)}$ has

(a) one solution
(b) two solutions
(c) no solution
(d) infinite number of solutions

36. The most general values of x for which $\sin x+\cos x=\min_{a\in R}\{1, a^2-4a+6\}$ are given by

(a) $2n\pi, n\in N$ (b) $2n\pi+\frac{\pi}{2}, n\in N$

(c) $n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{4}, n\in N$ (d) none of these

37. If $f(x)=\sin x+\cos x$, then the most general solutions of $f(x)=\left[f\left(\frac{\pi}{10}\right)\right]$ are (where $[x]$ is the greatest integer less than or equal to x.)

(a) $2n\pi+\frac{\pi}{2}, n\in I$ (b) $n\pi, n\in I$

(c) $2n\pi\pm\frac{2\pi}{3}, n\in I$ (d) none of these

38. If $x\in[0, 2\pi]$, $y\in[0, 2\pi]$ and $\sin x+\sin y=2$, then the value of $x+y$ is

(a) π (b) $\frac{\pi}{2}$

(c) 3π (d) none of these

39. The number of roots of the equation $x+2\tan x=\frac{\pi}{2}$ in the interval $[0, 2\pi]$ is

(a) 1 (b) 2
(c) 3 (d) infinite

40. If $x=X\cos\theta-Y\sin\theta$, $y=X\sin\theta+Y\cos\theta$ and $x^2+4xy+y^2=AX^2+BY^2$, $0\le\theta\le\frac{\pi}{2}$, then

(a) $\theta=\frac{\pi}{6}$ (b) $\theta=\frac{\pi}{4}$

(c) $A=-3$ (d) $B=1$

41. The number of solutions of the equation $\cos(\pi\sqrt{(x-4)})\cos(\pi\sqrt{x})=1$ is

(a) none (b) one
(c) two (d) more than two

42. The number of solutions of the equation

$$\sin\left(\frac{\pi x}{2\sqrt{3}}\right)=x^2-2\sqrt{3}x+4$$

(a) forms an empty set (b) is only one
(c) is only two (d) is greater than two

43. The solution of the equation $\log_{\cos x}\sin x+\log_{\sin x}\cos x=2$ is given by

(a) $x=2n\pi+\frac{\pi}{4}, n\in I$ (b) $x=n\pi+\frac{\pi}{2}, n\in I$

(c) $x=n\pi+\frac{\pi}{8}, n\in I$ (d) none of these

44. The general value of θ such that $\sin 2\theta=\frac{\sqrt{3}}{2}$ and $\tan\theta=\frac{1}{\sqrt{3}}$ is given by

(a) $n\pi+\frac{7\pi}{6}, n\in I$ (b) $n\pi\pm\frac{7\pi}{6}, n\in I$

(c) $2n\pi+\frac{7\pi}{6}, n\in I$ (d) none of these

45. Values of x and y satisfying the equation $\sin^7 y=|x^3-x^2-9x+9|+|x^3-x^2-4x+4|+\sec^2 2y+\cos^4 y$ are

(a) $x=1, y=n\pi, n\in I$
(b) $x=1, y=2n\pi+\frac{\pi}{2}, n\in I$
(c) $x=1, y=2n\pi, n\in I$
(d) none of the above

46. Number of real roots of the equation $\sec\theta+\operatorname{cosec}\theta=\sqrt{15}$ lying between 0 and 2π is

(a) 8 (b) 4
(c) 2 (d) 0

47. The solution of the equation $\sin^{10}x+\cos^{10}x=\frac{29}{16}\cos^4 2x$ is

(a) $x=\frac{n\pi}{4}+\frac{\pi}{8}, n\in I$ (b) $x=n\pi+\frac{\pi}{4}, n\in I$

(c) $x=2n\pi+\frac{\pi}{2}, n\in I$ (d) none of these

48. Solutions of the equation $|\cos x|=2[x]$ are (where [.] denotes the greatest integer function)

(a) nil (b) $x=\pm1$

(c) $x=\frac{\pi}{3}$ (d) none of these

49. The general solution of the equation $\sin^{100}x-\cos^{100}x=1$, is

(a) $2n\pi+\frac{\pi}{3}, n\in I$ (b) $n\pi+\frac{\pi}{2}, n\in I$

(c) $n\pi+\frac{\pi}{4}, n\in I$ (d) $2n\pi-\frac{\pi}{3}, n\in I$

50. The number of solutions of the equation $2^{\cos x}=|\sin x|$ in $[-2\pi, 2\pi]$ is

(a) 1 (b) 2
(c) 3 (d) 4

51. The general solution of the equation $2^{\cos 2x} + 1 = 3 \cdot 2^{-\sin^2 x}$ is

(a) $n\pi, n \in I$ (b) $n\pi + \pi, n \in I$

(c) $n\pi - \pi, n \in I$ (d) none of these

52. If $x \in (0, 1)$ the greatest root of the equation $\sin 2\pi x = \sqrt{2} \cos \pi x$ is

(a) 1/4 (b) 1/2

(c) 3/4 (d) none of these

53. If $\max\{5 \sin \theta + 3 \sin (\theta - \alpha)\} = 7$, then the set of possible values of α is ($\theta \in R$)

(a) $\left\{x : x = 2n\pi \pm \frac{\pi}{3}, n \in I\right\}$

(b) $\left\{x : x = 2n\pi \pm \frac{2\pi}{3}, n \in I\right\}$

(c) $\left[\frac{\pi}{3}, \frac{2\pi}{3}\right]$

(d) none of the above

54. The set of all x in $(-\pi, \pi)$ satisfying $|4 \sin x - 1| < \sqrt{5}$ is given by

(a) $x \in \left(-\frac{\pi}{10}, \pi\right)$ (b) $x \in \left(-\frac{\pi}{10}, \frac{3\pi}{10}\right)$

(c) $x \in \left(-\pi, \frac{3\pi}{10}\right)$ (d) $x \in (-\pi, \pi)$

55. The solution of the inequality $\log_{1/2} \sin x > \log_{1/2} \cos x$ in $[0, 2\pi]$ is

(a) $x \in \left(0, \frac{\pi}{2}\right)$ (b) $x \in \left(0, \frac{\pi}{8}\right)$

(c) $x \in \left(0, \frac{\pi}{4}\right)$ (d) none of these

56. The number of all possible triplets (x, y, z) such that $(x + y) + (y + 2z) \cos 2\theta + (z - x) \sin^2 \theta = 0$ for all θ is

(a) 0 (b) 1

(c) 3 (d) infinite

57. The number of solutions of $\tan(5\pi \cos \alpha) = \cot(5\pi \sin \alpha)$ for α in $(0, 2\pi)$ is

(a) 7 (b) 14

(c) 21 (d) 28

58. The number of solution (s) of the equation $\sin^3 x \cos x + \sin^2 x \cos^2 x + \sin x \cos^3 x = 1$ in the interval $[0, 2\pi]$ is/are

(a) no (b) one

(c) two (d) three

59. The most general values of x for which $\sqrt{3} \sin x - \cos x = \min_{\lambda \in R} \{2, e^2, \pi, \lambda^2 - 4\lambda + 7\}$ are given by

(a) $2n\pi, n \in I$

(b) $2n\pi + \frac{2\pi}{3}, n \in I$

(c) $n\pi + (-1)^n \frac{\pi}{4} + \frac{\pi}{6}, n \in I$

(d) $n\pi + (-1)^{n+1} \frac{\pi}{4} - \frac{\pi}{3}, n \in I$

60. The solution set of the inequality $\cos^2 \theta < \frac{1}{2}$ is

(a) $\left\{\theta : (8n + 1)\frac{\pi}{4} < \theta < (8n + 3)\frac{\pi}{4}, n \in I\right\}$

(b) $\left\{\theta : (8n - 3)\frac{\pi}{4} < \theta < (8n - 1)\frac{\pi}{4}, n \in I\right\}$

(c) $\left\{\theta : (4n + 1)\frac{\pi}{4} < \theta < (4n + 3)\frac{\pi}{4}, n \in I\right\}$

(d) none of the above

61. If $[y] = [\sin x]$ and $y = \cos x$ are two given equations, then the number of solutions is

([.] denotes the greatest integer function)

(a) 2 (b) 3

(c) 4 (d) infinitely many solutions

62. The number of solutions of the equation $1 + \sin x \sin^2\left(\frac{x}{2}\right) = 0$ in $[-\pi, \pi]$ is

(a) zero (b) 1

(c) 2 (d) 3

63. The number of solutions of the equation $|\cot x| = \cot x + \frac{1}{\sin x}$ $(0 \le x \le 2\pi)$ is

(a) 0 (b) 1

(c) 2 (d) 3

64. The real roots of the equation $\cos^7 x + \sin^4 x = 1$ in the interval $(-\pi, \pi)$ are

(a) $-\frac{\pi}{2}, 0$ (b) $-\frac{\pi}{2}, 0, \frac{\pi}{2}$

(c) $\frac{\pi}{2}, 0$ (d) $0, \frac{\pi}{4}, \frac{\pi}{2}$

65. Number of solutions of the equations $y = \frac{1}{3}[\sin x + [\sin x + [\sin x]]]$ and $[y + [y]] = 2 \cos x$, where [.] denotes the greatest integer function is

(a) 0 (b) 1

(c) 2 (d) infinite

Objective Questions Type II [One or more than one correct answers(s)]

In each of the questions below four choices of which are or more than one are correct. You have to select the correct answer(s) according.

1. The values of x satisfying $\sin^{-1} x + \sin^{-1}(1-x) = \cos^{-1} x$ are

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) 2

2. If $\frac{1}{2} < |x| < 1$, then which of the following are real?

(a) $\sin^{-1} x$ (b) $\tan^{-1} x$

(c) $\sec^{-1} x$ (d) $\cos^{-1} x$

3. $\sin^{-1} x > \cos^{-1} x$ holds for

(a) all values of x (b) $x \in \left(0, \frac{1}{\sqrt{2}}\right)$

(c) $x \in \left(\frac{1}{\sqrt{2}}, 1\right)$ (d) $x = 0.75$

4. If $6\sin^{-1}(x^2 - 6x + 8.5) = \pi$, then

(a) $x = 1$ (b) $x = 2$

(c) $x = 3$ (d) $x = 4$

5. Let $f(x) = e^{\cos^{-1} \sin\left(x + \frac{\pi}{3}\right)}$, then

(a) $f\left(\frac{8\pi}{9}\right) = e^{\frac{5\pi}{18}}$ (b) $f\left(\frac{8\pi}{9}\right) = e^{\frac{13\pi}{18}}$

(c) $f\left(-\frac{7\pi}{4}\right) = e^{\frac{\pi}{12}}$ (d) $f\left(-\frac{7\pi}{4}\right) = e^{\frac{11\pi}{12}}$

6. If $\alpha \le \sin^{-1} x + \cos^{-1} x + \tan^{-1} x \le \beta$, then

(a) $\alpha = 0$ (b) $\beta = \pi/2$

(c) $\alpha = \pi/4$ (d) $\beta = \pi$

7. The greatest and least values of $(\sin^{-1} x)^3 + (\cos^{-1} x)^3$ are

(a) $\frac{\pi^3}{32}$ (b) $-\frac{\pi^3}{8}$

(c) $\frac{7\pi^3}{8}$ (d) $\frac{\pi}{2}$

8. The value of $\tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$ is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) π (d) 0

9. The solution of the equation $\sin[2\cos^{-1}\{\cot(2\tan^{-1} x)\}] = 0$ are

(a) ± 1 (b) $1 \pm \sqrt{2}$

(c) $-1 \pm \sqrt{2}$ (d) none of these

10. α, β and γ are the angles given by $\alpha = 2\tan^{-1}(\sqrt{2} - 1)$, $\beta = 3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ and $\gamma = \cos^{-1}\left(\frac{1}{3}\right)$, then

(a) $\alpha > \beta$ (b) $\beta > \gamma$

(c) $\gamma > \alpha$ (d) none of these

11. Indicate the relation which is true

(a) $\tan|\tan^{-1} x| = |x|$ (b) $\cot|\cot^{-1} x| = |x|$

(c) $\tan^{-1}|\tan x| = |x|$ (d) $\sin|\sin^{-1} x| = |x|$

12. $\cos^{-1}\left(\sqrt{\frac{a-x}{a-b}}\right) = \sin^{-1}\left(\sqrt{\frac{x-b}{a-b}}\right)$ is possible, if

(a) $a > x > b$

(b) $a < x < b$

(c) $a = x = b$

(d) $a > b$ and x, takes any value

13. $\theta = \tan^{-1}(2\tan^2\theta) - \tan^{-1}\left\{\left(\frac{1}{3}\right)\tan\theta\right\}$, if

(a) $\tan\theta = -2$ (b) $\tan\theta = 0$

(c) $\tan\theta = 1$ (d) $\tan\theta = 2$

14. If the numerical value of $\tan\left\{\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right\}$ is $\frac{a}{b}$, then

(a) $a + b = 23$ (b) $a - b = 11$

(c) $3b = a + 1$ (d) $2a = 3b$

15. If $\text{cosec}^{-1} x = \sin^{-1}\left(\frac{1}{x}\right)$, then x may be

(a) 1 (b) $-\frac{1}{2}$

(c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

16. $2\cot^{-1} 7 + \cos^{-1}\left(\frac{3}{5}\right)$ is equal to

(a) $\cot^{-1}\left(\frac{44}{117}\right)$ (b) $\text{cosec}^{-1}\left(\frac{125}{117}\right)$

(c) $\tan^{-1}\left(\frac{4}{117}\right)$ (d) $\cos^{-1}\left(\frac{44}{125}\right)$

17. If the equation $\sin^{-1}(x^2 + x + 1) + \cos^{-1}(\lambda x + 1) = \frac{\pi}{2}$ has exactly two solutions, then λ cannot have the integral value

(a) –1 (b) 0

(c) 1 (d) 2

18. The value(s) of x satisfying the equation $\sin^{-1}|\sin x| = \sqrt{\sin^{-1}|\sin x|}$ is/are given by (n is any integer)

(a) $n\pi - 1$ (b) $n\pi$

(c) $n\pi + 1$ (d) $2n\pi + 1$

19. If $\tan^{-1} y = 4\tan^{-1} x$, then y is infinite, if

(a) $x^2 = 3 + 2\sqrt{2}$ (b) $x^2 = 3 - 2\sqrt{2}$

(c) $x^4 = 6x^2 - 1$ (d) $x^4 = 6x^2 + 1$

20. If $\cos^{-1} x = \tan^{-1} x$, then

(a) $x^2 = (\sqrt{5} - 1)/2$

(b) $x^2 = (\sqrt{5} + 1)/2$

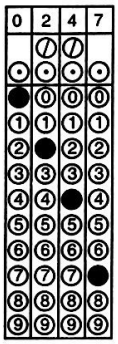
(c) $\sin(\cos^{-1} x) = (\sqrt{5} - 1)/2$

(d) $\tan(\cos^{-1} x) = (\sqrt{5} - 1)/2$

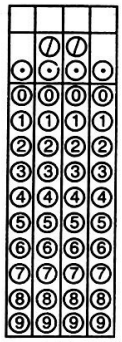
Numerical Grid-Based Problems

Solve the following problems and mark your response against their respective grids. Write your answer in the top row of the grid and darken the concerned numbers in the respective columns.

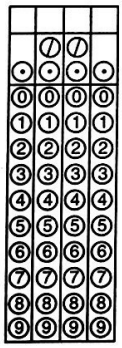
For example. If answer of a question is 0247, then



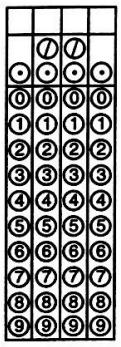
1. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, then the value of $2500(x + y + z) - \frac{216}{(x^3 + y^3 + z^3)}$ must be



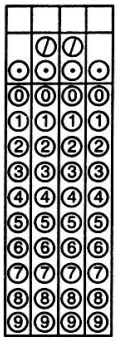
2. The value of $\frac{1}{\pi}\left\{216 \sin^{-1}\left(\sin\frac{7\pi}{6}\right) + 27\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + 28\tan^{-1}\left(\tan\frac{5\pi}{4}\right) + 200\cot^{-1}\left(\cot\left(\frac{-\pi}{4}\right)\right)\right\}$ must be



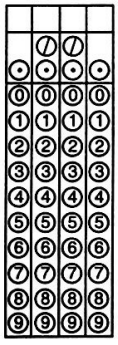
3. If $S = \sum_{r=1}^{50} \tan^{-1}\left(\frac{2r}{2 + r^2 + r^4}\right)$, then the value of $2550 \cot S$ must be



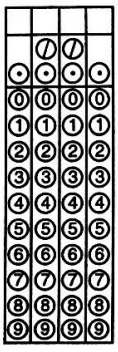
4. If $\tan^{-1}(x + 1) + \tan^{-1} x + \tan^{-1}(x - 1) = \tan^{-1} 3$, then the value of (for $x < 0$) $500x^4 + 270x^2 + 997$ must be



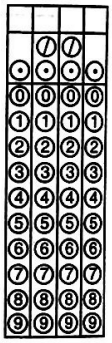
5. If $\lambda = \cos^4[\tan^{-1}\{\sin(\cot^{-1} 5)\}]$, then the value of $3645\,\lambda$ must be



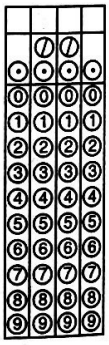
6. If $\theta = \cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18$, then the value of $81 \cot^4 \theta$ must be



7. If $\lambda = \tan\left(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right)$, then the value of $2890\,\lambda^2$ must be



8. If $\sin^{-1} x + \sin^{-1} y = \pi$ and, if $x = \lambda y$, then the value of $39^{2\lambda} + 5^{\lambda}$ must be



Matrix-Match Type

Given below are Matching Type Questions, with two columns (each having some items) each . Each item of Column I has to be matched with the items of Column II, by encircling the correct match(es).

NOTE An item of Column I can be matched with more than one items of Column II. All the items of Column II have to be matched.

1. **Observe the following columns :**

	Column I		Column II
(A)	If principal values of $\sin^{-1}\left(-\frac{1}{2}\right)+\tan^{-1}(\sqrt{3})$ and $\cos^{-1}\left(-\frac{1}{2}\right)$ are λ and μ respectively, then	(P)	$\lambda+\mu=\frac{\pi}{2}$
(B)	If Principal values of $\sin^{-1}\left(\sin\frac{7\pi}{6}\right)$ and $\cos^{-1}\left\{-\sin\left(\frac{5\pi}{6}\right)\right\}$ are λ and μ respectively, then	(Q)	$\mu-\lambda=\frac{\pi}{2}$
(C)	If principal values of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ and $\sin^{-1}\left\{\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\right\}$ are λ and μ respectively, then	(R)	$\lambda+\mu=-\frac{\pi}{6}$
		(S)	$\mu-\lambda=\frac{5\pi}{6}$
		(T)	$\lambda+\mu=\frac{5\pi}{6}$

(A) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ (B) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ (C) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ

2. **Observe the following columns :**

	Column I		Column II
(A)	If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$, then	(P)	$x=y=z$
		(Q)	$xyz\geq 3\sqrt{3}$
(B)	If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\frac{\pi}{2}$, then	(R)	$x+y+z=xyz$
(C)	If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\frac{\pi}{2}$ and $x+y+z=\sqrt{3}$, then	(S)	$xyz\leq\frac{1}{3\sqrt{3}}$
		(T)	$xy+yz+zx=1$

(A) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ (B) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ (C) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ

3. **Observe the following columns :**

	Column I		Column II
(A)	If $2\tan^{-1}(2x+1)=\cos^{-1}(-x)$, then x is	(P)	$-\frac{1}{\sqrt{2}}$
		(Q)	0
(B)	If $2\cos^{-1}x=\sin^{-1}(2x\sqrt{(1-x^2)})$, then x is	(R)	$\frac{1}{\sqrt{2}}$
		(S)	$\frac{\sqrt{3}}{2}$
(C)	If $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$, then x is	(T)	1

(A) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ (B) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ (C) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ

Answers

Objective Questions Type I [Only one correct answer]

1.	(a)	**2.**	(d)	**3.**	(b)	**4.**	(a)	**5.**	(a)	**6.**	(b)	**7.**	(c)	**8.**	(c)	**9.**	(d)	**10.**	(b)
11.	(c)	**12.**	(c)	**13.**	(c)	**14.**	(c)	**15.**	(a)	**16.**	(b)	**17.**	(c)	**18.**	(d)	**19.**	(b)	**20.**	(a)
21.	(a)	**22.**	(a)	**23.**	(c)	**24.**	(b)	**25.**	(a)	**26.**	(a)	**27.**	(a)	**28.**	(a)	**29.**	(c)	**30.**	(c)
31.	(b)	**32.**	(a)																
				33.	(d)	**34.**	(c)	**35.**	(c)	**36.**	(c)	**37.**	(a)	**38.**	(a)	**39.**	(c)	**40.**	(b)
41.	(b)	**42.**	(b)	**43.**	(a)	**44.**	(d)	**45.**	(b)	**46.**	(b)	**47.**	(a)	**48.**	(a)	**49.**	(b)	**50.**	(d)
51.	(a)	**52.**	(c)	**53.**	(a)	**54.**	(b)	**55.**	(c)	**56.**	(d)	**57.**	(b)	**58.**	(a)	**59.**	(b)	**60.**	(c)
61.	(d)	**62.**	(a)	**63.**	(c)	**64.**	(b)	**65.**	(a)										

Objective Questions Type II [One or more than one correct answer(s)]

1. (a, b)	**2.** (a, b, d)	**3.** (c, d)	**4.** (b, d)	**5.** (b, c)
6. (a, d)	**7.** (a, c)	**8.** (c, d)	**9.** (a, b, c)	**10.** (b, c)
11. (a, b, d)	**12.** (a, b)	**13.** (a, b, c)	**14.** (a, b, c)	**15.** (a, c, d)
16. (a, b, d)	**17.** (a, c, d)	**18.** (a, b, c)	**19.** (a, b, c)	**20.** (a, c)

Linked-Comprehension Type

Passage 1 **1.** (d) **2.** (d) **3.** (d) **4.** (c) **5.** (c)

Passage 2 **1.** (a) **2.** (d) **3.** (c) **4.** (d) **5.** (b)

Passage 3 **1.** (d) **2.** (c) **3.** (a) **4.** (d) **5.** (b)

Numerical Grid-Based Problems

1.	7	4	2	8	**2.**	0	1	3	9	**3.**	2	5	5	2	**4.**	1	7	6	7
5.	3	3	8	0	**6.**	6	5	6	1	**7.**	0	4	9	0	**8.**	1	5	2	6

Matrix-Match Type

1. A → (Q, T); B → (P, S); C → (Q, R)
2. A → (Q, R); B → (S, T); C → (P, S, T)
3. A → (Q); B → (R, S, T); C → (P, R)